

**Violation of supersymmetric equivalence in R parity
violating couplings**

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Abstract

In this paper we consider the violation of supersymmetric equivalence among the R parity violating couplings λ_{ijk} caused by widely split chiral supermultiplets. The calculations have been done for two specific models of supersymmetry breaking: a) heavy SQCD models and b) 2-1 models. We find that if $\lambda'_{2jk} \approx g$ and $\lambda'_{1jk} \approx e$ then the violation of SUSY equivalence is of the order of 5-6 % in heavy SQCD models. On the other hand if $\lambda'_{3jk} \approx g$ and $\lambda_{ijk} \approx e$ then the violation of of SUSY equivalence is of the order of 9.4 % in 2-1 models.

Introduction

Low energy supersymmetry (SUSY) provides an attractive solution to the naturalness problem and perhaps also the hierarchy problem of the SM [1]. The search for SUSY will therefore constitute one of the major activities of all future high energy colliders. After the discovery of SUSY the next task will be to measure the masses and couplings of the sparticles with high precision. If SUSY is exact then the couplings that are related by means of a supersymmetric transformation must be equal to all orders in perturbation theory. However if SUSY is softly broken then although these couplings will be equal at the tree level, radiative corrections will introduce small splitting between them. Such violation of supersymmetric equivalence among the couplings increases logarithmically with the heavy sparticle mass scale (M) [2]. Therefore if some of the sparticles are very heavy then although they decouple from most low energy processes, they nevertheless produce a non-decoupling effect through radiative corrections. In fact it could happen that some of the sparticles are very heavy and inaccessible at the future high energy colliders, but we can probe their masses and couplings indirectly by precision measurement of such SUSY breaking effects.

Implications of exact SUSY on R parity breaking couplings

Consider the R parity violating couplings that break lepton number but conserves baryon number [3]. Such effects are given by the following Lagrangians:

$$\begin{aligned}
 L_1 = & \lambda_{ijk} [\tilde{\nu}_L^i \bar{e}_R^k e_L^j + \tilde{e}_L^j \bar{e}_R^k \nu_L^i + (\bar{e}_R^k)^* (\tilde{\nu}_L^i)^c e_L^j \\
 & - (i \leftrightarrow j)] + h.c.
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 L_2 = & \lambda'_{ijk} [\tilde{\nu}_L^i \bar{d}_R^k d_L^j + \tilde{d}_L^j \bar{d}_R^k \nu_L^i + (\bar{d}_R^k)^* (\tilde{\nu}_L^i)^c d_L^j \\
 & - \tilde{e}_L^i \bar{d}_R^k u_L^j - \tilde{u}_L^j \bar{d}_R^k e_L^i - (\bar{d}_R^k)^* (\tilde{e}_L^i)^c u_L^j] + h.c.
 \end{aligned} \tag{2}$$

In this paper we shall consider the violation of SUSY equivalence among the couplings of L_1 that are related by supersymmetric transformations. The reason being λ_{ijk} unlike λ'_{ijk} are amenable to precision measurements since they are free from QCD related uncertainties. Secondly in many models the squarks turn out to be very heavy and they lie in the Tev range. For such models the couplings λ'_{ijk} cannot be directly measured at colliders operating in the few hundred Gev range. Note that the couplings associated with the three terms $\tilde{\nu}_L^i \bar{d}_R^k d_L^j$, $\tilde{d}_L^j \bar{d}_R^k \nu_L^i$ and $(\tilde{d}_R^k)^* (\bar{\nu}_L^i)^c d_L^j$ are related by supersymmetric transformations. If SUSY is exact then their couplings should be equal and this common coupling has been denoted in the above by λ_{ijk} . However if SUSY is broken at some high energy scale M then the three couplings will differ from each other at a low energy scale $\mu \ll M$. In broken SUSY we shall denote the three couplings by $\lambda_{ijk}^{(1)}$, $\lambda_{ijk}^{(2)}$ and $\lambda_{ijk}^{(3)}$. They satisfy the boundary condition $\lambda_{ijk}^1(M) = \lambda_{ijk}^2(M) = \lambda_{ijk}^3(M) = \lambda_{ijk}$ at the mass scale M for heavy sparticles. The splitting between the couplings at μ arises because the heavy sparticles decouple at M . If the members of each supermultiplet have the same mass and the radiative corrections due to all of them are taken into account then the three couplings will evolve in the same manner and there will be no difference between them at μ . The splitting between the couplings at μ due to widely split supermultiplets can therefore be estimated by considering only the loop diagrams that involve one or more heavy sparticles. In this paper the splitting between the three couplings will be calculated for two specific models of SUSY breaking: a) heavy SQCD models and b) 2-1 models.

Radiative corrections to λ_{ijk} in heavy SQCD models

In heavy SQCD models like gauge mediated SUSY breaking [4] the colored squarks and the gluinos are much heavier than the corresponding ordinary particles. Therefore the wavefunction and vertex renormalization constants for $\lambda_{ijk}^{(1)}$, $\lambda_{ijk}^{(2)}$ and $\lambda_{ijk}^{(3)}$ that involve at least one heavy sparticle can arise from λ'_{ijk} only. Consider the vertex renormalization constants associated with the three couplings. Vertex renormalization diagrams for $\lambda_{ijk}^{(1)}$ that contain at least one heavy squark line can arise if L_2 contains a $\tilde{\nu} q \bar{q}$ vertex, a $\tilde{q}^* \bar{e} R q$

vertex and a $\tilde{q}\bar{q}e_L$ vertex. However we find that although L_2 contains a $\tilde{\nu}^i\bar{d}_R^k d_L^j$ vertex and a $\tilde{u}_L^j\bar{d}_R^k e_L^i$ vertex it does not contain a $\tilde{q}\bar{e}_R q'_L$ vertex. Hence there is no relevant vertex correction diagram for λ_{ijk}^1 . Similarly there are also no vertex correction diagrams for $\lambda_{ijk}^{(2)}$ and $\lambda_{ijk}^{(3)}$ since L_2 does not contain $\tilde{q}\bar{e}_R q'_L$ and $\tilde{e}_R^* \bar{q} q'$ vertices. In order that the ratios $\frac{\lambda_{ijk}^{(1)}}{\lambda_{ijk}^{(2)}}$ and $\frac{\lambda_{ijk}^{(1)}}{\lambda_{ijk}^{(3)}}$ measure the effects of SUSY breaking only we must ensure that the same set of family indices are involved in $\lambda^{(1)}$, $\lambda^{(2)}$ and $\lambda^{(3)}$. For example let us consider the case of $i = 2$ and $j = k = 1$. In evaluating the renormalization constants we shall use the \overline{MS} scheme and keep only the leading log terms and the terms that remain finite in the limit $\frac{\mu}{M} \rightarrow 0$ where μ is the mass scale for light sparticles. Consider now the wavefunction renormalization constants associated with $\lambda_{211}^{(1)}$, $\lambda_{211}^{(2)}$ and $\lambda_{211}^{(3)}$. We find that only the self energy diagrams for e_L^1 , ν_L^2 and ν_L^{2c} involves a heavy squark line. The wavefunction renormalization constants are given by

$$\sqrt{Z}_{e_L^1} = 1 - \frac{N_c}{64\pi^2} \sum_{j,k} |\lambda'_{1jk}|^2 (\ln \frac{M^2}{\mu^2} - 1). \quad (3)$$

and

$$\sqrt{Z}_{\nu_L^2} = \sqrt{Z}_{\nu_L^{2c}} = 1 - \frac{N_c}{64\pi^2} \sum_{jk} |\lambda'_{2jk}|^2 (\ln \frac{M^2}{\mu^2} - 1). \quad (4)$$

where $N_c = 3$ is the number of colors and μ is the renormalization mass scale which we shall assume to be around 100 GeV. Using the boundary condition $\lambda_{211}^1(M) = \lambda_{211}^2(M) = \lambda_{211}^3(M)$ we then get

$$\frac{\lambda_{211}^{(2)}(\mu)}{\lambda_{211}^{(1)}(\mu)} = 1 - \frac{N_c N_g^2}{64\pi^2} (\bar{\lambda}_2'^2 - \bar{\lambda}_1'^2) (\ln \frac{M^2}{\mu^2} - 1). \quad (5)$$

and

$$\frac{\lambda_{211}^{(3)}(\mu)}{\lambda_{211}^{(1)}(\mu)} = 1 - \frac{N_c N_g^2}{64\pi^2} \bar{\lambda}_2'^2 (\ln \frac{M^2}{\mu^2} - 1). \quad (6)$$

In the above $N_g^2 \bar{\lambda}'_1{}^2 \equiv \sum_{j,k} |\lambda'_{1jk}|^2$ and $N_g^2 \bar{\lambda}'_2{}^2 \equiv \sum_{j,k} |\lambda'_{2jk}|^2$. N_g is the number of fermion generations. In models of gauge mediated SUSY breaking if the light sparticles have a mass of few hundred GeV then the squarks usually lie in the 1 TeV mass range. For $\bar{\lambda}'_1{}^2 = e^2$ and $\bar{\lambda}'_2{}^2 = g^2$ we find that

$$\lambda_{211}^{(1)}(\mu) : \lambda_{211}^{(2)}(\mu) : \lambda_{211}^{(3)}(\mu) = 1 : 1 - .051 : 1 - .066 \quad (7)$$

So in this case the violation of SUSY equivalence is 5.1% between $\lambda_{211}^{(1)}$ and $\lambda_{211}^{(2)}$ and 6.6% between $\lambda_{211}^{(3)}$ and $\lambda_{211}^{(1)}$. The splittings between $\lambda_{211}^{(1)}$, $\lambda_{211}^{(2)}$ and $\lambda_{211}^{(3)}$ in heavy SQCD models is therefore quite large to be detectable through precision measurements of λ_{ijk} at future e^+e^- colliders.

The constraints on λ'_{ijk} derived from low energy phenomenology that are given in standard references [5] scale with the sparticle mass scale. The couplings λ'_{ijk} that appear in the above radiative corrections involve one heavy squark field in the TeV range. The present experimental bounds on such couplings are therefore very weak. The reason is not hard to see. Sparticles with a mass of around 1 TeV decouple from low energy processes and therefore their R parity violating couplings can be quite large. The values of $\bar{\lambda}'_1$ and $\bar{\lambda}'_2$ assumed above are therefore consistent with the experimental bounds and in fact are much more restrictive. If we had used the present experimental bounds on λ'_{ijk} the splitting between $\lambda_{211}^{(1)}$, $\lambda_{211}^{(2)}$ and $\lambda_{211}^{(3)}$ would have been much larger.

Radiative corrections to λ_{ijk} in 2-1 models

In 2-1 models [6] all sparticles belonging to the first and second generations are very heavy. So in this case we also have to include the radiative corrections from loop diagrams that contain one heavy slepton line belonging to the first or second generation. Since only the sleptons of the third generation are light consider the following terms of L_1 :

$$\begin{aligned} L_1 = & \lambda_{3jk}^{(1)} \tilde{\nu}_L^3 \bar{e}_R^k e_L^j + \lambda_{i3k}^{(2)} \tilde{e}_L^3 \bar{e}_R^k \nu_L^i \\ & + \lambda_{ij3}^{(3)} \tilde{e}_R^{3*} \bar{\nu}_L^{ic} e_L^j + \dots \end{aligned} \quad (8)$$

Note that since λ_{ijk} must be antisymmetric in the first two indices it is not possible to compare $\lambda_{3jk}^{(1)}$ with $\lambda_{ik3}^{(2)}$ as a measure of SUSY breaking. The only possibilities are to compare $\lambda_{3j3}^{(1)}$ with $\lambda_{3j3}^{(3)}$ or $\lambda_{i33}^{(2)}$ with $\lambda_{i33}^{(3)}$. Let us consider for example $\lambda_{323}^{(1)}$ and $\lambda_{323}^{(3)}$. As before there is no vertex correction associated with $\lambda_{323}^{(1)}$ or $\lambda_{323}^{(3)}$ arising from λ'_{ijk} . It can also be shown that there is no vertex correction for $\lambda_{323}^{(1)}$ or $\lambda_{323}^{(3)}$ arising from λ_{ijk} that involves a heavy slepton line. The renormalizations of $\lambda_{323}^{(1)}$ and $\lambda_{323}^{(3)}$ from M to μ are therefore given solely by their respective wavefunction renormalization constants. We find that in 2-1 models the wavefunction renormalization of e_R^3 and ν_L^{3c} are given by

$$\sqrt{Z}_{e_R^3} = [1 - \frac{1}{32\pi^2} (2|\lambda_{123}|^2 + |\lambda_{233}|^2 + |\lambda_{133}|^2) (\ln \frac{M^2}{\mu^2} - 1)]. \quad (9)$$

and

$$\begin{aligned} \sqrt{Z}_{\nu_L^{3c}} = & [1 - \frac{N_c}{64\pi^2} \sum_{j=1}^3 \sum_{k=1}^2 |\lambda'_{3jk}|^2 (\ln \frac{M^2}{\mu^2} - 1) \\ & - \frac{1}{64\pi^2} \sum_{j=1}^2 \sum_{k=1}^2 |\lambda_{3jk}|^2 (\ln \frac{M^2}{\mu^2} - 1)] \end{aligned} \quad (10)$$

Note that in $\sqrt{Z}_{\nu_L^{3c}}$ the contribution from λ'_{ijk} is summed over k from 1 to 2 since only the squarks of first two generations are heavy. Whereas in the contribution from λ_{ijk} the sum over j and k are determined by the antisymmetry of λ_{ijk} in the first two indices and the fact that only the sparticles of the third generation are light.

$$\begin{aligned} \frac{\lambda_{323}^{(1)}(\mu)}{\lambda_{323}^{(3)}(\mu)} = & 1 + \frac{1}{64\pi^2} [N_c \sum_{j=1}^3 \sum_{k=1}^2 |\lambda'_{3jk}|^2 + \sum_{j=1}^2 \sum_{k=1}^2 |\lambda_{3jk}|^2 \\ & - 2(2|\lambda_{123}|^2 + |\lambda_{233}|^2 + |\lambda_{133}|^2)] (\ln \frac{M^2}{\mu^2} - 1) \end{aligned} \quad (11)$$

In 2-1 models if the light sparticles are in the few hundred GeV range then the heavy sparticles of the first two generations can lie in the 10 TeV range without violating the low energy constraints arising from FCNC. To get a numerical estimate of the violation

of SUSY equivalence between $\lambda_{323}^{(1)}$ and $\lambda_{323}^{(3)}$ let us assume that $\lambda'_{3jk} \approx g$ and $\lambda_{1jk} \approx e$. We then find that $\frac{\lambda_{323}^{(1)}(\mu)}{\lambda_{323}^{(3)}(\mu)} \approx 1 + .094$ which is again large enough to be detectable through precision measurements of λ_{ijk} at future e^+e^- colliders.

Conclusions

In this paper we have considered the violation of SUSY equivalence among the couplings $\lambda_{ijk}^{(1)}$, $\lambda_{ijk}^{(2)}$ and $\lambda_{ijk}^{(3)}$ that are related by means of supersymmetric transformations. We have computed the splitting between these couplings in the context of heavy SQCD models and 2-1 models. We find that if $\lambda'_{2jk} \approx g$ and $\lambda'_{1jk} \approx e$ then the violation of SUSY equivalence is of the order of 5-6% in heavy SQCD models. On the other hand if $\lambda'_{3jk} \approx g$ and $\lambda_{ijk} \approx e$ then the violation of SUSY equivalence can be as large as 9.4% in 2-1 models. In either model the splittings caused by widely split chiral supermultiplets are quite large to be detectable through precision measurements of $\lambda_{ijk}^{(1)}$, $\lambda_{ijk}^{(2)}$ and $\lambda_{ijk}^{(3)}$ at future e^+e^- colliders.

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